Suggested Solutions to:
Regular Exam, Spring 2015
Industrial Organization
May 27, 2015

This version: June 10, 2015

Question 1: Informational advertising and strategic incentives

Part (a)

In order to solve for the equilibrium prices, suppose $\lambda_1 > 0$ and $\lambda_2 > 0$ and study the profit maximization problem of firm 1. The FOC (w.r.t. p_1) associated with this problem is given by

$$\begin{aligned} & \frac{\partial E\pi_1}{\partial p_1} \\ = \;& \lambda_1 \left[1 - \lambda_2 + \lambda_2 \left(\frac{p_2 - p_1 + \tau}{2\tau}\right) - \frac{\lambda_2}{2\tau}(p_1 - c)\right] \\ = \;& 0 \end{aligned}$$

or

$$2\tau(1-\lambda_2) + \lambda_2(p_2 - p_1 + \tau) - \lambda_2(p_1 - c) = 0$$

or

$$(p_2 - 2p_1 + \tau + c)\lambda_2 + 2\tau(1-\lambda_2) = 0$$

or

$$\begin{aligned} (2p_1 - p_2)\lambda_2 &= (\tau + c)\lambda_2 + 2\tau(1-\lambda_2) \\ &= 2\tau - (\tau - c)\lambda_2 \end{aligned}$$

or

$$2p_1 - p_2 = \frac{2\tau}{\lambda_2} - \tau + c. \quad (1)$$

By symmetry of the problem, the FOC for firm 2 can be written as

$$2p_2 - p_1 = \frac{2\tau}{\lambda_1} - \tau + c. \quad (2)$$

Before proceeding, and for the purpose of drawing the figure later, we note that firm 1's and firm 2's best reply functions can be written as

$$p_1 = \frac{\tau}{\lambda_2} + \frac{c - \tau + p_2}{2} \overset{\text{def}}{=} R_1$$

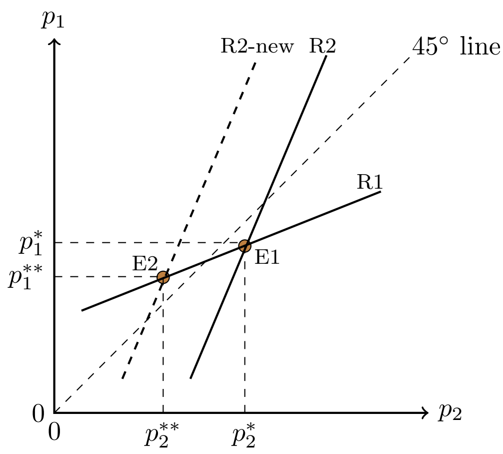

Figure 1: Illustration of the second-stage equilibrium (for given values of $\lambda_1 > 0$ and $\lambda_2 > 0$) and of the comparative statics exercise. See the solutions to Q1a.

and

$$p_2 = \frac{\tau}{\lambda_1} + \frac{c - \tau + p_1}{2} \overset{\text{def}}{=} R_2$$

Now write (1) and (2) on matrix form:

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} \frac{2\tau}{\lambda_2} - \tau + c \\ \frac{2\tau}{\lambda_1} - \tau + c \end{bmatrix}$$

So, using Cramer's rule, we obtain

$$\begin{aligned} p_1^* &= \frac{2\left(\frac{2\tau}{\lambda_2} - \tau + c\right) + \frac{2\tau}{\lambda_1} - \tau + c}{3} \\ &= \frac{2\tau}{3}\left(\frac{1}{\lambda_1} + \frac{2}{\lambda_2}\right) + c - \tau \end{aligned}$$

and

$$p_2^* = \frac{2\tau}{3}\left(\frac{2}{\lambda_1} + \frac{1}{\lambda_2}\right) + c - \tau.$$

- It follows from the analysis above that the firms' stage 2 choice variables are strategic complements (for the best replies are upward-sloping).

- In Figure 1, the equilibrium is illustrated as the intersection of the firms' best replies (E1).
- Figure 1 also illustrates the effect of an increase in λ_1: Firm 2's best reply shifts in the north-west direction (but firm 1's best reply is unaffected). This, in turn, moves the equilibrium along firm 1's best reply, from E1 to E2. The new equilibrium therefore involves *lower* prices for *both* firms. So the effect of an increase in λ_1 on p_1^* and p_2^* is negative: Both prices go down.

Part (b)

The effects are as indicated below:

$$\begin{aligned}&\frac{\partial \pi_1}{\partial \lambda_1}\\ =\ &\underbrace{(p_1^* - c)\left[1 - \lambda_2 + \lambda_2\left(\frac{p_2^* - p_1^* + \tau}{2\tau}\right)\right] - a_1\Phi'(\lambda_1)}_{\text{direct effect}}\\ &+\underbrace{\frac{\lambda_1\lambda_2(p_1^* - c)}{2\tau}\frac{\partial p_2^*}{\partial \lambda_1}}_{\text{strategic effect}}. \qquad (3)\end{aligned}$$

The **direct effect** is the effect on profit that is due directly to the increase in advertising: It will (i) raise the advertising cost and it will (ii) increase sales and therefore revenues (the latter is a benefit and the former, obviously, a cost).

The **strategic effect** (or indirect effect) is the effect on profit that is due to the change in the rival's price that the increase in the (own) advertising level induces. That is, if firm 1 increases λ_1, then this can be observed by firm 2 before firm 2 chooses its price. Moreover, the new level of λ_1 will change the economic environment at the price-setting stage and, in particular, firm 2's incentives. This may lead to a lower or a higher firm 2 price, which in turn may be good or bad for firm 1's profit. From eq. (1) it is clear that (assuming that $p_1^* > c$) the strategic effect has the same sign at the derivative $\partial p_2^*/\partial\lambda_1$. That is, if a higher advertising level for firm 1 made firm 2's equilibrium price lower, then this would provide a reason for firm 1 to, all else equal, advertise less (i.e., then the strategic effect would be negative).

Part (c)

Now set $c = 0$ and $a_1 = a_2 = a$, as stated in the question. Also note (from above) that

$$\frac{\partial p_2^*}{\partial \lambda_1} = -\frac{4\tau}{3\lambda_1^2} < 0.$$

Setting $\frac{\partial \pi_1}{\partial \lambda_1} = 0$ in (3) and imposing symmetry, we have

$$p^*\left(1 - \frac{\lambda^*}{2}\right) + \frac{(\lambda^*)^2 p^*}{2\tau}\frac{\partial p_2^*}{\partial \lambda_1} = a\Phi'(\lambda^*)$$

or

$$\begin{aligned}a\Phi'(\lambda^*) &= p^*\left(1 - \frac{\lambda^*}{2}\right) - \frac{2p^*}{3}\\ &= \frac{p^*}{6}(6 - 3\lambda^* - 4)\\ &= \frac{p^*}{6}(2 - 3\lambda^*)\end{aligned}$$

Also note (from above) that

$$p^* = \frac{2\tau}{3}\left(\frac{2}{\lambda^*} + \frac{1}{\lambda^*}\right) + c - \tau = \frac{\tau(2 - \lambda^*)}{\lambda^*}.$$

So the above equality becomes

$$a\Phi'(\lambda^*) = \frac{\tau(2 - 3\lambda^*)(2 - \lambda^*)}{6\lambda^*}.$$

That is, the function that we were asked to specify is given by

$$f(\lambda^*) = \frac{\tau(2 - 3\lambda^*)(2 - \lambda^*)}{6\lambda^*}.$$

Question 2: Cournot competition with asymmetric firms

Part (a)

At an equilibrium, firm i chooses a quantity that maximizes its profit, given the equilibrium actions of the other forms. If this quantity is positive, which we suppose it is, then the following first-order condition must be satisfied:

$$\begin{aligned}&\frac{\partial \pi_i(q_1, \ldots, q_n)}{\partial q_i}\\ =\ &P\left(\sum_{j=1}^{n} q_i\right) + q_i P'\left(\sum_{j=1}^{n} q_i\right) - C_i'(q_i)\\ =\ &0.\end{aligned}$$

Rewriting yields

$$P\left(\sum_{j=1}^{n} q_i\right) - C_i'(q_i) = -q_i P'\left(\sum_{j=1}^{n} q_i\right)$$

or

$$\frac{P\left(\sum_{j=1}^{n} q_i\right) - C_i'(q_i)}{P\left(\sum_{j=1}^{n} q_i\right)} = -\frac{q_i P'\left(\sum_{j=1}^{n} q_i\right)}{P\left(\sum_{j=1}^{n} q_i\right)}.$$

The left-hand side of the above expression is the Lerner index for firm i, L_i. The right-hand side can be rewritten as follows:

$$-\frac{q_i P'\left(\sum_{j=1}^n q_i\right)}{P\left(\sum_{j=1}^n q_i\right)} = -\frac{\left[\sum_{j=1}^n q_i\right] P'\left(\sum_{j=1}^n q_i\right) q_i}{P\left(\sum_{j=1}^n q_i\right)\left[\sum_{j=1}^n q_i\right]} = \frac{1}{\eta}\alpha_i.$$

We thus have

$$L_i = \frac{\alpha_i}{\eta},$$

which we were supposed to show.

Why is L_i increasing in α_i?

- The Lerner index for firm i, L_i, measures the extent to which the firm charges a price above its marginal cost — so the firm's market power. When the firm's market share is larger (at the equilibrium, due to a lower production cost), the firm is closer to a situation where it is a monopolist. A monopolist optimally chooses a smaller quantity (which corresponds to charging a higher price) than a Cournot duopolist does, because its output equals total market output and an increase in the quantity will therefore have a bigger (negative) impact on the market price.

Why is L_i decreasing in η?

- The parameter η measures the price elasticity of demand, that is, the extent to which the consumers' willingness to buy the good is sensitive to price changes. When η goes up, the consumers become more sensitive to price changes. It is therefore harder for the firm to profitably increase its price above its marginal cost.

Part (b)

From above and from the question we have that the Lerner index for firm i, at the equilibrium, can be written as

$$L_i = \frac{\alpha_i}{\eta}.$$

Multiplying each side by α_i, we have

$$\alpha_i L_i = \frac{\alpha_i^2}{\eta}.$$

This equality holds for each one of the n firms in the market. Adding up across firms, we obtain

$$\sum_{i=1}^n \alpha_i L_i = \sum_{i=1}^n \frac{\alpha_i^2}{\eta} \stackrel{\text{def}}{=} \frac{I_H}{\eta},$$

where $\mathrm{I}_H \stackrel{\text{def}}{=} \sum_{i=1}^n \alpha_i^2$ is the Herfindahl index. The above equality is identical to eq. (4) in the question.

We are also asked to explain the reasoning behind the following claim: *The result in (4) supports the idea that market concentration is associated with market power.* The Herfindahl index is meant to measure the extent of concentration in a market — that is, the extent to which production and sales in the market tend to be done mostly by a small number of firms (as opposed to production and sales being spread out fairly equally across the firms in the market). This index will be smaller if, for any given number of firms, the market shares are more similar to each other. The index will also be smaller if, given equal market shares, the number of firms in the market goes up.

The result stated in eq. (4) in the question means that, under certain assumptions, the Herfindahl index will be proportional to one particular measure of aggregate market power, namely our "average Lerner index,"as defined in the question. So the result provides a theoretical justification for thinking of concentration (as measured by the Herfindahl index) as being positively linked with market power (as measured by the average Lerner index). The main assumption that is required for this result is that the market outcome can be described by the equilibrium of the Cournot model (which assumes quantity setting and simultaneous moves, for example).

Part (c)

First derive an expression for firm i's equilibrium output.

- Firm i solves (taking all others' output as given):

$$\max_{q_i \geq 0} \left(a - c_i - b\sum_{j=1}^n q_j\right) q_i.$$

- If $q_i^* > 0$, firm i's FOC holds:

$$a - c_i - b\sum_{j=1}^n q_j - bq_i = 0.$$

- Rewriting the FOC, we get $q_i = \frac{a-c_i}{b} - Q$, where $Q \stackrel{\text{def}}{=} \sum_{j=1}^n q_j$.

- Add up across all n firms:

$$\sum_{j=1}^{n} q_j = \frac{na - \sum_{j=1}^{n} c_j}{b} - nQ$$
$$\Leftrightarrow \quad Q^* = \frac{na - \sum_{j=1}^{n} c_j}{b\,(n+1)}.$$

- Plugging Q^* back in q_i expression yields firm i's equilibrium output:

$$\begin{aligned} q_i^* &= \frac{a - c_i}{b} - Q^* \\ &= \frac{a - nc_i + \sum_{j \neq i} c_j}{b\,(n+1)}, \end{aligned} \tag{4}$$

  which we were asked to derive.

Next we are asked to argue formally that a firm gains by being relatively efficient (i.e., by having a cost parameter that is low relative to the rivals' cost parameters). To this end, let us calculate firm i's equilibrium profit.

- First calculate firm i's price-cost margin:

$$\begin{aligned} p^* - c_i &\stackrel{\text{def}}{=} a - bQ^* - c_i \\ &= \frac{a - nc_i + \sum_{j \neq i} c_j}{n+1}. \end{aligned}$$

- Hence firm i's profit at the equilibrium is

$$\pi_i^* = (p^* - c_i)\, q_i^* = b\,(q_i^*)^2\,.$$

- It is clear from the above expression that firm i's equilibrium profit is increasing in its equilibrium quantity. This quantity, in turn, is increasing in the rivals' costs and decreasing in the own cost (we see this from (4)) — in that sense we can say that a firm gains by being relatively efficient.